LAWS OF MOTION

Force and Inertia

A force is something which tends to change the state of rest or motion of a body.

Inertia is the tendency of any body to change its state of rest or motion. It is measured by the mass of the body. The larger is the mass of a body, the larger is its inertia to change in its state of motion or rest.

Example: A passenger getting down from a moving bus, falls in the direction of the motion of the bus. This is an example for

(a) moment of inertia (b) second law of motion
(c) third law of motion (d) inertia of rest
(e) inertia of motion

Sol. (e) On touching the ground his feet come to rest but the remaining body continues to move due to inertia of motion.

Newton's Law of Motion

Newton's laws of motion give the (future) positions and velocities of a body if the force on a body of mass m is known and its position and velocity at any given instant of time are also given.

Newton's First Law Of Motion states that a body in a state of rest or of uniform motion continues to be in this state, unless acted upon by an external force. Mathematically, it states that if $F_{Net} = 0$, the acceleration of the body will be zero.

Newton's Second Law Of Motion states that the rate of change of momentum of a body is equal to the impressed force on the body. Mathematically, it can be written as

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv)$$

Example: A particle is travelling along a straight line OX. The distance x (in meters) of the particle from O at a time t is given by $x = 37 + 27t - t^3$ where t is time in seconds. The distance of the particle from O when it comes to rest is

(A) 81 m (B) 91 m
(C) 101 m (D) 111 m

Sol. (B) The velocity of the particle is

$$v = \frac{dx}{dt} = 27 - 3t^2 = 0$$

When $t = 3$ sec.

$$\therefore \quad x = (37 + 27 \times 3 - 3^3)\ m = (37 + 81 - 27)\ m = 91\ m$$

Momentum

Momentum of a body is defined as the product of mass and velocity of the body. i.e.

$$p = m\,v$$

Thus, if the mass of the moving system is constant,

$$F = m\frac{dv}{dt} = ma$$

However, if the mass is variable,

$$F = m\frac{dv}{dt} + v\frac{dm}{dt}$$

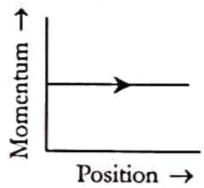
As a consequence of this law, if

$$F = 0, \quad \frac{dp}{dt} = 0 \text{ or } p = \text{Constant}$$

PARAGRAPH BASED QUESTIONS

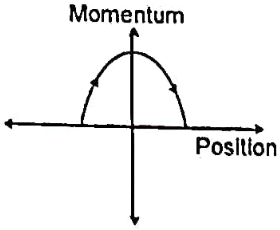
Paragraph for Examples Nos. (i) to (iii):

Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is x(t) vs. p(t) curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative.

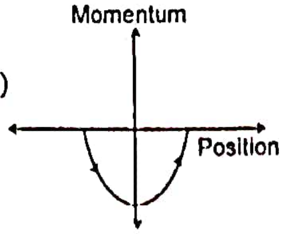


Example : The phase space diagram for a ball thrown vertically up from ground is

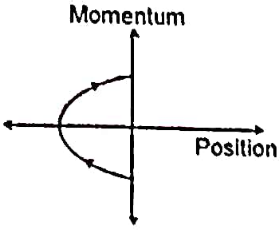
(A)



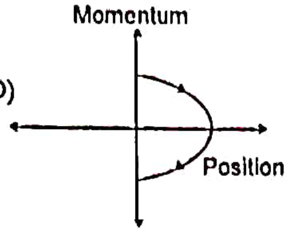
(B)



(C)



(D)



Sol. (D) With the passage of time, position is positive and increases; momentum is upwards (positive) and (decreases) with time in magnitude.
Both position (vector) and momentum (vector) are then negative. Momentum increases in magnitude and position magnitude decreases to zero.

Force

A force is any kind of a push or pull which changes or tends to change the state of rest or of uniform motion of a body. Forces do not always give rise to motion. Mathematically, a force is defined by the equation

$$F = ma$$

i.e. it is the product of the mass of the body and the acceleration produced in the body by the force. It is measured in units of newton (N). One newton is a force which when acting on a body of mass 1 kg produces in it an acceleration of 1 m/s^2.

A force is a vector quantity and has a magnitude as well as a direction.

Example: A cricket ball of mass 0.25 kg with speed 10 m/s collides with a bat and returns with same speed within 0.01 second. The force acted on bat is

(a) 25 N (b) 50 N
(c) 250 N (d) 500 N

Sol. (d) $F = \frac{\Delta P}{\Delta t} = \frac{2\ mv}{\Delta t}$

$= \frac{2 \times 0.25 \times 10}{0.01} N$

or $F = 500$ N

Impulse

The impulse of a constant force $\vec{F}$ is defined as F Δt, applied during time interval Δt.

The impulse of a variable force is defined as $\int F\,dt$

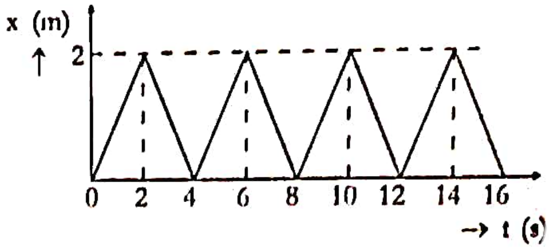
According to the impulse-momentum theorem

$$\int F\Delta t = \Delta p = mv - mu$$

where the impulse changes the velocity of the body of mass m from u to v.

Impulsive force is defined as a force which is variable in size and which acts for a very short time only.

Example: The figure shows the position-time (x-t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is



(a) 0.2 Ns (b) 0.4 Ns
(c) 0.8 Ns (d) 1.6 Ns

Sol. Impulse = change in momentum
$= m(\Delta v) = 0.4(1 + 1)$
$= 0.8$ Ns

Thus (c) is the correct answer.

Law of Conservation of Linear Momentum

If no net external force acts on a body, then its momentum remains constant. This is the *Law of Conservation of Linear Momentum.* This law is of great importance in the study of various phenomena in Physics. It has various applications, particularly in the study of collisions and interactions between different particles.

Equilibrium of Concurrent Forces

Equilibrium of a particle means that the net external force acting on the particle is zero. Thus, according to the Newton's first law of motion, a particle is in equilibrium if it is either at rest or in uniform motion.

If two forces $\vec{F}_1$ and $\vec{F}_2$ act on a particle, it will be in equilibrium, if $\vec{F}_1 = -\vec{F}_2$, so that $\vec{F}_1 + \vec{F}_2 = 0$.

A set of forces are said to be **concurrent** if they pass through a common point but do not have the same line of force.

A body is in equilibrium under concurrent forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \ldots$ if $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots = 0$

$\Rightarrow F_{1x} + F_{2x} + F_{3y} + \ldots = 0$

$F_{1y} + F_{2y} + F_{3y} + \ldots = 0$

$F_{1z} + F_{2z} + F_{3z} + \ldots = 0$

According to **Newton's Third Law of Motion,** *to every action there is an equal and opposite reaction.* Rocket propulsion is an example of Newton's third law of motion and the principle of conservation of momentum. In a rocket, a stream of burnt gases is exhausted at a very high temperature and pressure through a nozzle. These escape at high velocities. *The reaction of the escaping gases provides the necessary thrust to the rocket which is then propelled in the opposite direction.*

As the gases are ejected at a nearly constant velocity with respect to the rocket, the rate of change of momentum is nearly constant. However, due to rapid burning of fuel, the mass of the rocket-fuel system decreases. Hence acceleration of the rocket keeps on increasing.

***Example*:** A stationary body of mass 3 kg explodes into three equal pieces. Two of the pieces fly off in two mutually perpendicular directions, one with a velocity of $3\hat{i}$ ms^{-1} and the other with a velocity of $4\hat{j}$ ms^{-1}. If the explosion occurs in 10^{-4} s, the average force acting on the third piece in newton is

(a) $(3\hat{i}+4\hat{j})\times10^{-4}$ (b) $(3\hat{i}-4\hat{j})\times10^{-4}$

(c) $(3\hat{i}+4\hat{j})\times10^{4}$ (d) $-(3\hat{i}+4\hat{j})\times10^{4}$

Sol. Since the total momentum is conserved, the third piece has momentum $=1\times-(3\hat{i}+4\hat{j})$ kg ms^{-1}

Also, impulse = Average force × time

$$\therefore \text{ Average force } = \frac{\text{Impulse}}{\text{time}}$$

$$= \frac{\text{Change in momentum}}{\text{time}}$$

$$= \frac{-(3\hat{i}+4\hat{j})\text{ kg ms}^{-1}}{10^{-4}\text{ s}}$$

$$=-(3\hat{i}+4\hat{j})\times10^{4}\text{ N}$$

Thus (d) is the correct answer.

Friction

Friction is defined as a force resisting the relative motion between two solid surfaces, fluid layers or material elements sliding against each other.

The coefficient of friction, μ, describes the ratio of the force of friction between two bodies and the force of pressing them together. Its value depends on the two materials involved.

Static and Kinetic Friction

Static friction is the friction between two solid objects which are not in motion relative to each other. The force of static friction must be overcome by an applied force before an object can move. The maximum possible force of static friction between two surfaces before one slides over the other is given by

$F_{max} = \mu_s F_n$

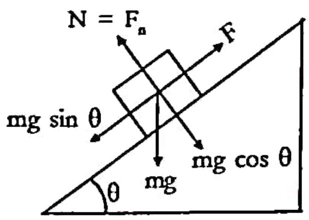
where μ_s is called the coefficient of static friction between the surfaces and F_n is the normal component of the force acting on them. If the applied force $F < F_{max}$, it is opposed by the frictional force of equal magnitude and opposite direction. Thus, the value of frictional force varies between zero to F_{max}. As soon as the sliding occurs, the friction between the surfaces is no longer static and is called the **kinetic friction.**

The coefficient of friction between two surfaces when they are moving relative to each other is denoted by μ_k. Usually $\mu_k < \mu_s$ for any pair of substances.

Angle of Friction

The coefficient of static friction can also be defined in terms of the maximum angle at which one of the surfaces is inclined before which the other item (surface) placed on it will begin to slide. This angle is called the angle of friction or repose angle. For this angle θ,

$\tan\theta = \mu_s$.



Laws of Friction

1. The force of friction is always in a direction opposite to the direction of motion.
2. The force of friction is directly proportional to the applied force.
3. The force of friction is independent of the area and shape of the surfaces in contact.
4. The force of kinetic friction is independent of the sliding velocity. It remains constant even if the body is accelerated.

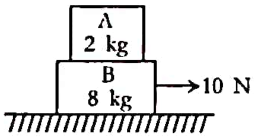
Rolling Friction

When a wheel ball or solid cylinder rolls freely over a surface, rolling friction comes into play. The main source of friction in a rolling phenomenon is the dissipation of energy involved in the deformation of the objects (e.g., soil). The force of rolling friction is expressed as

$f_r = \mu_r F_n$

μ_r is generally much smaller than μ_s.

Example: Block A of mass 2 kg is placed over block B of mass 8 kg. The combination is placed over a rough horizontal surface. Coefficient of friction between B and the floor is 0.5. Coefficient of friction between A and B is 0.4. A horizontal force of 10 N is applied on block B. The force of friction between A and B is .....



(1) zero (2) 50 N

(3) 40 N (4) 100 N

Sol. (1) Net frictional force between the block B and the surface is

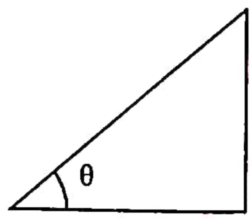
$F = \mu R = 0.5 \times 10 \times 10 = 50$ N

Applied force is 10 N which is less than the frictional force.

$\therefore$ System is at rest and there is no force of friction between A and B.

Example: A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define $N = 10\mu$, then N is

Sol. (5)



Given, $mg(\sin\theta + \mu\cos\theta) = 3\,mg(\sin\theta - \mu\cos\theta)$

$\Rightarrow \sin 45° + \mu\cos 45° = 3(\sin 45° - \mu\cos 45°)$

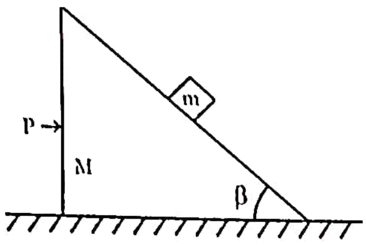
$$\Rightarrow \frac{1}{\sqrt{2}}(1+\mu) = 3\times\frac{1}{\sqrt{2}}(1-\mu)$$

Thus $1+\mu = 3-3\mu$

or $\mu = 0.5$

$\therefore N = 10\mu = 5$

Example: Two wooden blocks of masses M and m are placed on a smooth horizontal surface as shown in figure. If a force P is applied to the system as shown in figure such that the mass m remains stationary with respect to block of mass M, then the magnitude of the force P is



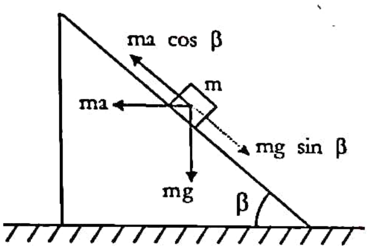
(1) $(M + m)g \tan \beta$ (2) $g \tan \beta$
(3) $mg \cos \beta$ (4) $(M + m)g \operatorname{cosec} \beta$

Sol. (1) Acceleration of the system of blocks,

$$a = \frac{P}{M+m} \quad \text{...(i)}$$

Also, for equilibrium,

$$mg \sin \beta = ma \cos \beta$$



$$\Rightarrow \quad a = g \tan \beta = \frac{P}{M+m} \quad \text{[Using (i)]}$$

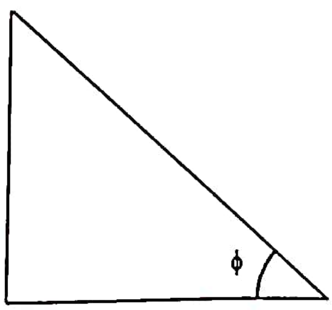
Thus $P = (M + m)g \tan \beta$

Example: The upper half of an inclined plane with an angle of inclination ϕ, is smooth while the lower half is rough. A body starting from rest at the top of the inclined plane comes to rest at the bottom of the inclined plane. Then the coefficient of friction for the lower half is

(1) $2 \tan \phi$ (2) $\tan \phi$
(3) $2 \sin \phi$ (4) $2 \cos \phi$

Sol. (1) For the upper half of the inclined plane, the acceleration

$$a_1 = g \sin \phi$$



Velocity of the body at the end of the upper half is

$$v_1 = \sqrt{2a_1\left(\frac{L}{2}\right)} = \sqrt{2g \sin \phi \times \frac{L}{2}}$$

$$= \sqrt{g L \sin \phi}$$

For the lower half plane, the acceleration

$$a_2 = g(\sin \phi - \mu \cos \phi)$$

where μ is the coefficient of friction.

Using $v^2 - u^2 = 2as$

$$0 - g L \sin \phi = 2g(\sin \phi - \mu \cos \phi) \times \frac{L}{2}$$

$$\Rightarrow \quad 2g L \sin \phi = L g m \cos \phi$$

Hence $\mu = 2 \tan \phi$

Dynamics of Uniform Circular Motion

A particle moving in a circle of radius r with a uniform speed v experiences a force F, called centripetal force which acts towards the centre of the circle and along its radius.

$$F = \frac{mv^2}{r} = mr\omega^2$$

where ω is the angular velocity of the particle.

The acceleration of the particle called the *centripetal acceleration* is

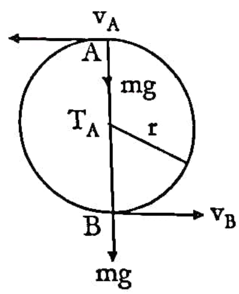
$$a = \frac{v^2}{r} = r\omega^2$$

Its radial and angular velocities are related as

$$v = r\,\omega$$

Motion in a Vertical Circle

Consider a particle of mass m moving in a circle of radius r as shown in the figure below :



As the particle moves from B to A, it loses its kinetic energy but gains gravitational energy.

(i) At the highest point A,

$$\frac{mv_A^2}{r} = T_A + mg$$

When the particle just reaches the point A,

$$T_A = 0, \; v_A = v_C$$

$$\therefore \quad v_c = \sqrt{gr}$$

v_c denotes the critical speed of the particle at the point A.

Further, $\frac{1}{2}m(v_B^2 - v_A^2) = mg \times 2r$

$$\Rightarrow \quad v_B = \sqrt{5gr}$$

The condition for the body to complete one revolution is that $T_A > 0$, which implies

$$v_B = \sqrt{5gr}$$

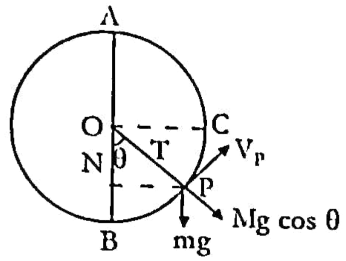
(ii) At the lowest point,

$$\frac{mv_B^2}{r} = T_B - mg$$

$\Rightarrow$ $T_B = 6\ mg$

Velocity and Tension at any Point

At any point (see figure)



$$T - mg\cos\theta = \frac{mv_P^2}{r}$$

$$T = mg\left(\frac{r-h}{r}\right) + \frac{mv_P^2}{r}$$

$$= \frac{m}{r}\left[v_P^2 + g(r-h)\right]$$

Further $v_P^2 = v_B^2 - 2gh$

$= 5\ gr - 2\ gr\ (1 - \cos\theta)$

$= 3\ gr + 2\ gr\cos\theta$

So that

$$v_C = \sqrt{3\ gr} \text{ and}$$

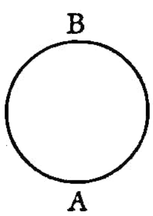
$$v_A = \sqrt{gr}$$

Example: A particle is moving with a constant speed v in a circle. What is the magnitude of average velocity after half rotation?

(a) $2v$ (b) $2\frac{v}{\pi}$

(c) $\frac{v}{2}$ (d) $\frac{v}{2\pi}$

Sol. (b)



Suppose the particle travels from A to B to complete half rotation.

Time taken $t = \frac{\pi r}{v}$

Distance covered = 2r

$\therefore$ Average velocity $= \frac{2r}{\pi r} \times v = \frac{2v}{\pi}$

Example: A particle is moving with a uniform speed v in a circular path of radius r with the centre at O. When the particle moves from a point P to Q on the circle such that $\angle POQ = \theta$, then the magnitude of the change in velocity is

(A) $2v\ \sin(2\theta)$ (B) Zero

(C) $2v\sin\left(\frac{\theta}{2}\right)$ (D) $2v\cos\left(\frac{\theta}{2}\right)$

Sol. (C) $\left|\Delta\vec{v}\right| = \sqrt{v^2 + v^2 - 2v^2\cos\theta}$

$$= 2\ v\ \sin\frac{\theta}{2}$$

Centripetal Force and its Applications

A particle moving in a circle or any motion in a curved path is an accelerated motion, and requires a force directed toward the center of the circle or curvature of the path. This force is called the centripetal force (or center seeking force). For a particle of mass m moving in a circle with uniform speed V, the centripetal force is given by

$$F_{centripetal} = m\frac{V^2}{r}$$

The centripetal acceleration is given by

$$\frac{F_{centripetal}}{m} = \frac{V^2}{r}$$

An application of the centripetal force, is drying (partially) the washed clothes in a washing machine. Another important application is the motion of a satellite in outer space. The force of gravity constantly pulls the satellite towards the center of the earth. By keeping the distance just right, the satellite is made to stay in the same circular orbit, requiring no extra energy or other external force to keep it on a constant path. Swinging of a mass on a string requires tension in the string due to centripetal force, and the mass will travel off in a tangential straight line if the string breaks. The centripetal force is proportional to the square of the velocity, implying that a doubling of speed will require four times the centripetal force to keep the motion in a circle. If the centripetal force must be provided by friction alone on a curved path, an increase in speed could lead to an unexpected skid if friction is insufficient.

Banking of Curved Roads

Roads are banked on curves, (the outer portion of the road is raised with respect to the inner portion) to provide the centripetal force needed by the vehicles as they move around the curve. The angle of banking depends on the radius of curvature of the curve as well as the speed at which the vehicle is to be driven on the curve. It is given by the equation

$$\tan\theta = \frac{v^2}{rg}$$